

1. Show mathematically whether each of the following sequences is arithmetic, geometric, or neither. If it is arithmetic or geometric, find an explicit equation to model the sequence.

a. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$ Neither $\frac{1}{3} - \frac{1}{2} \neq \frac{1}{4} - \frac{1}{3}$

$\frac{\frac{1}{3}}{\frac{1}{2}} \neq \frac{\frac{1}{4}}{\frac{1}{3}}$

b. -1, 3, -9, 27, ...

Geometric

$r = \frac{3}{-1} = \frac{-9}{3} = \frac{27}{-9} = \boxed{-3}$ $a_n = -1(-3)^{n-1}$

c. 5, 8, 11, 14, ...

Arithmetic

$d = 8 - 5 = 11 - 8 = 14 - 11 = \boxed{3}$ $a_n = 5 + 3(n-1)$

2. Find the 14th term of each sequence. Hint: use the arithmetic equation to help you do this. . .

a. the sequence in part (b) of number 1

$a_{14} = -1(-3)^{14-1} = \boxed{1,594,323}$

b. the sequence in part (c) of number 1

$a_{14} = 5 + 3(14-1)$
 $= 5 + 39$
 $= \boxed{44}$

3. Find the missing terms of the arithmetic sequence below algebraically. Show work.

$$\{-9.5, -3.25, \underline{\quad}, \underline{\quad}, 9.25, 15.5, 21.75, 28\}$$

\downarrow $\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \downarrow$
 a_1 $\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad a_4$

$$28 = 3 + 4d$$

$$25 = 4d$$

$$6.25 = d$$

4. In the sequence in problem (3), which term of the sequence is 603?

$$603 = -9.5 + 6.25(n-1)$$

$$612.5 = 6.25(n-1)$$

$$98 = n-1$$

$$99 = n$$

99th term

5. $\frac{4428675}{131072}$ is which term of the below sequence? Solve algebraically and show work.

$$\frac{4428675}{131072} = \frac{800}{800} \left(\frac{3}{4}\right)^{n-1}$$

$$\frac{177147}{4194304} = \left(\frac{3}{4}\right)^{n-1}$$

$$\log_{3/4} \frac{177147}{4194304} = 11^{\text{th}} \text{ term}$$

$\{800, 600, 450, \dots\}$
 $r = \frac{600}{800} = \frac{3}{4} = 0.75$

5. Suppose you drop a bounce ball from a certain height and measure the height of each bounce. The sequence of heights is geometric. Find an equation for a sequence that generates the bounce heights of the ball if the height of the 5th bounce is 9 ft and the height of the 7th bounce is 5 ft.

$$a_1 \quad \quad \quad \frac{1209}{a_5} \quad \quad \quad \frac{5}{a_7}$$

$$5 = 9r^2$$

$$\frac{5}{9} = r^2$$

$$\frac{\sqrt{5}}{3} = r \approx 0.745$$

$$a_1 = 9 \cdot \frac{3}{\sqrt{5}} \cdot \frac{3}{\sqrt{5}} \cdot \frac{3}{\sqrt{5}} \cdot \frac{3}{\sqrt{5}}$$

$$= 9 \cdot \frac{81}{25}$$

$$= \frac{729}{25}$$

$$a_n = \frac{729}{25} \left(\frac{\sqrt{5}}{3}\right)^{n-1}$$